

Department of Environmental Quality Land Quality Division



WYOMING

Handbook of Approved Sampling and Statistical Methods for Evaluation of Revegetation Success on Wyoming Coal Mines

by

Carol Bilbrough DEQ-LQD

&

Shay Howlin Western Ecosystems Technology, Inc.

TABLE OF CONTENTS

Introduction.....	3
Section 1: Sampling Methods for Revegetation Success Evaluation	4
(A) Sampling Plans.....	4
(B) Sample Units and Reference Areas.....	4
(C) Maps.....	4
(D) Vegetation descriptions.....	4
(E) Species inventory	5
(F) General Comments on vegetation sampling methods.....	5
(G) Sampling for vegetation cover	6
(H) Sampling for total herbaceous production on grazingland	8
(I) Sampling for total herbaceous production on cropland	9
(J) Sampling for shrub density	9
(K) Measuring tree replacement	10
(L) Sampling trees for reforestation.....	10
(M) Sampling for species diversity and composition.....	10
Section 2: Submittal of Information for Revegetation Success Evaluation.....	11
(A) Methods description.....	11
(B) Maps.....	11
(C) Vegetation Descriptions	11
(D) Seeding information.....	11
(E) Species inventory	11
(F) Acreage information	11
(G) Results of vegetation cover and production analyses using reference areas	11
(H) Results of vegetation cover and production analyses using technical standards	12
(I) Results of the shrub density standard analysis	13
(J) Results of the shrub goal assessment	14
(K) Results of the species diversity and composition assessment.....	14
(L) Results of the cropland standard analysis	15
(M) Results of the tree replacement standard assessment.....	15
(N) Demonstration of mitigation wetlands restoration.....	15
(O) Results of the reforestation standard analysis	15
(P) Results from fish & wildlife habitat, recreational, industrial/commercial or residential, and mitigation wetland analyses	16
(Q) Appendices.....	16
Section 3: Statistical Procedures for Revegetation Success Evaluation	17
Flow Charts for Statistical Procedures.....	19
Example Data and Calculations	21
One-sample procedures.....	22
Two-sample procedures	28
Statistical Tables	35

Introduction

This document conforms with requirements by the Office of Surface Mining that the Regulatory Authority approve and publish statistically valid sampling techniques for measuring revegetation success (CFR 30 816.116(a)(1)). The methods described below are the sampling and statistical methods approved by the Land Quality Division for evaluating revegetation success according to the standards contained in Chapter 4, Section 2(d)(ii) of the Coal Rules and Regulations.

The document is divided into three sections; sampling methods, requirements for submittal, and statistical methods.

Section 1: Sampling Methods for Revegetation Success Evaluation

(A) Sampling Plans

The sampling plans for revegetation success evaluation shall be submitted to the Division prior to the field sampling season for review and approval prior to implementation, unless otherwise approved by the Administrator.

(B) Sample Units and Reference Areas

The sampling plan shall include a table listing the proposed revegetation sample units, the reclaimed plant community type, acreages for each unit, postmining land use, and the reference area type and their acreages.

(C) Maps

The applicant shall map Sample Units and Reference Areas, in accordance with the following requirements:

- (I) The base map shall be an aerial photograph or topographic map at a scale approved by the Administrator.
- (II) The map shall include:
 - (1.) The boundaries of each Sample Unit and Reference Area;
 - (2.) The boundaries of all sampled shrub goal or shrub standard patches;
 - (3.) The location of each sampled point and transect direction, when used;
 - (4.) The location of wildlife habitat features;
 - (5.) The general location of trees;
 - (6.) The location, identity, and orientation of each photograph that is provided with the descriptions of the Sample Units and Reference Areas; and
 - (7.) An identification number, legend and title.
- (III) The Sample Units, Shrub Patches, and Reference Areas may be mapped any time the ground is clear of snow, but must be field checked and verified prior to the sampling.
- (IV) If one or more of the species defined in Chapter 1 as “species lacking creditable value” is estimated to comprise more than 25% of the relative vegetation cover on two or more contiguous acres, that acreage shall be identified on the map.

(D) Vegetation descriptions

Each revegetation success evaluation study shall contain descriptions of each Sample Unit and each Reference Area. The descriptions shall include:

- (I) The vegetation composition;

- (II) The major species in each life form;
- (III) The characteristic topography, including overall slope and aspect;
- (IV) A summary of any quantitative, semi-quantitative, and qualitative vegetation information gathered;
- (V) Information on present and historical weed treatment; and
- (VI) Three-inch by five-inch (or larger) color images showing the general features of each Sample Unit and Reference Area.

(E) Species inventory

The applicant shall compile a separate inventory of all plants species observed within each Sample Unit and Reference Area, in accordance with the following requirements:

- (I) Plant species shall be listed:
 - (1.) By life form (Chapter 1, Section 2(cb)); Shrubs and subshrubs remain as separate life forms, although they are combined into a single category for the diversity standard;
 - (2.) By scientific binomial (with reference to the botanic key used);
 - (3.) By common name; and
 - (4.) Identified as a species native to North America, or as an introduced species.
- (II) The species inventory shall be field checked and updated three times during the growing season each year of the sampling program to include species that grow at different times. The inventory will be reported separately for each Sample Unit and Reference Area. The species inventory will not be compared to any quantitative, semi-quantitative or qualitative criteria.
- (III) The applicant shall note the names and field locations of:
 - (1.) Any herbarium samples collected;
 - (2.) Any plant species or habitat of special concern at the time of sampling; and
 - (3.) Any species not previously recorded in Wyoming or is located outside its known range.

(F) General Comments on vegetation sampling methods

- (I) The type of comparison used to evaluate revegetation success (Reference Area or Technical Standard) does not affect which options for sampling methods are available for each Sample Unit. Methods and requirements for sample adequacy calculations for Sample Units and Reference Areas are described in the statistics

section of this document. Requirements specific to Reference Areas are not applicable if a Technical Standard is used to evaluate a revegetation success parameter.

- (II) The location of all sample points shall be chosen by random or systematic procedures, in accordance with the following considerations:
 - (1.) The same procedures shall be used for choosing sample locations in each Sample Unit and corresponding Reference Area;
 - (2.) Sample locations shall be readily located in the field, such as by pacing from easily identifiable landmarks or by use of Global Positioning System equipment;
 - (3.) Sample locations that fall within a barren or otherwise seemingly atypical area shall not be discarded only because the area is barren or atypical; although; and
 - (4.) Areas obviously disturbed by human activities shall be excluded from sampling.

- (III) Sampling dates for a specific vegetation parameter in each sampling year shall be determined in accordance with the following considerations:
 - (1.) Normal sampling dates are after June 1st and before September 1st, provided that actual sampling dates are based on regional and seasonal plant phenology and considerations for specific parameters, such as peak production. If an earlier or later sampling time frame is necessary, the operator shall provide justification for the selected time frame, prior to sampling, to assist the Administrator in determining if that time frame is appropriate;
 - (2.) Sampling of revegetation sample units and reference areas shall occur within a three-week period. If a different sampling time period is necessary, the operator shall provide justification, prior to sampling, to assist the Administrator in a determination;
 - (3.) Trees and shrubs may be sampled any time of the year, provided the species can be identified; and
 - (4.) Sample Units and Reference Areas must be sampled during the same time period in the same year.

(G) Sampling for vegetation cover

Cover sampling shall be conducted using quadrats or point intercept transects. Sample units and reference areas should be treated similarly prior to and during sampling (e.g. grazing or no grazing).

- (I) The following cover parameters shall be estimated:
 - (1.) Percent absolute vegetation cover by species;

- (2.) Percent absolute vegetation cover of all species combined;
 - (3.) Percent absolute cover of lichens and moss;
 - (4.) Percent absolute bare ground;
 - (5.) Percent absolute rock cover;
 - (6.) Percent absolute litter cover; and
 - (7.) Percent absolute total ground cover (vegetation + litter + rock + lichens + moss);
- (II) Sample size for cover, including minimum sample size and sample adequacy calculations, shall be based on the criteria in Section 3 of this document describing statistical methods.
- (III) Species defined in Chapter 1 of the Coal Rules and Regulations as “species lacking creditable value” shall be separately recorded. Species lacking creditable value shall be deleted from the comparison of total absolute vegetation cover between the Sample Unit and Reference Area.
- (IV) When the operator uses cover quadrats, each quadrat shall cover a minimum of 0.5, but not more than 1.0, square meter, unless an alternate quadrat size will allow for more accurate cover measurement. An alternate quadrat size must be approved by the Administrator prior to initiation of cover sampling:
- (1.) The sampler shall provide a method to determine percent cover by areal extent at the 1% level;
 - (2.) Each quadrat shall be considered as one observation for statistical calculations;
 - (3.) Sample Units and Reference Areas shall use the same quadrat size each sample year. Quadrat size may vary between years;
 - (4.) If multiple quadrats are used along a transect, a minimum of two quadrats shall be arranged evenly along each transect. Quadrat size shall be within the size range specified in (IV) above, and cover values averaged among quadrats within each transect; and
 - (5.) Each quadrat (or the average of quadrats along a transect) shall be considered one observation for statistical calculations.
- (V) When the operator uses point-intercept transects, all transects shall be at least 50 meters in length unless an alternate transect length will allow for more accurate cover measurement. Each transect shall have a minimum of 100 points per transect, unless otherwise approved by the Administrator prior to the initiation of sampling. The sample points shall be spaced at equal increments along each transect. The minimum measurement precision shall not be greater than 1%;

- (1.) Transects shall be the same length and have the same number of-sample points along each transect in a Sample Unit and associated Reference Area each year. Transect length may vary between years;
- (2.) The orientation of each transect shall be randomly established;
- (3.) If a transect runs out of a given Sample Unit or Reference Area, the sampler shall select a new random orientation, at the point where the transect leaves the Sample Unit or Reference Area, which will return the transect to the same Sample Unit or Reference Area;
- (4.) Point samples shall be determined using an ocular sighting device, such as a laser or cross-hairs, or a sharpened rod(s) in a fixed frame projected vertically downward to the sample location on the transect; and
- (5.) Each transect shall be considered as one observation for statistical calculations.

(H) Sampling for total herbaceous production on grazingland

Sampling for total herbaceous production of species shall be conducted in accordance with the following requirements:

- (I) The production sampling methods shall be the same for the Sample Unit and corresponding Reference Area, but the methods may vary between years;
- (II) Sample size for production, including minimum sample size and sample adequacy calculations, shall be based on the criteria in Section 3 of this document describing statistical methods;
- (III) Each production quadrat shall cover an minimum area of 0.5 square meter unless an alternate quadrat size will reduce the variance in the production values among samples. Alternate quadrat sizes shall be approved by the Administrator, prior to initiation of sampling;
- (IV) The standing crop biomass of all herbaceous species shall be harvested in each quadrat and clipped as close as practicable to the ground, excluding:
 - (1.) All “species lacking creditable value” as defined in Chapter 1, and
 - (2.) Full shrubs, succulents, annual forbs, *Yucca* species, cushion plants, mosses, lichens and trees shall be excluded from harvest.
- (V) The applicant shall report the specific clipping heights and drying methods used;
- (VI) All biomass shall be reported to the nearest 0.1 gram. At a minimum, data shall be reported in grams per square meter; and
- (VII) All production sampling shall be taken from within grazing exclosures. All exclosures shall be established on or before April 15th, or as soon as field

conditions permit. Enclosure size must exceed the quadrat size to accommodate both the sample quadrat and a buffer area. Extra enclosure locations shall be established to provide replacement locations, if needed. A fence used to control livestock grazing is considered one type of enclosure.

(I) Sampling for total herbaceous production on cropland

Sampling for total annual herbaceous production on lands which carry the cropland land use designations shall be conducted in accordance with one of the following methods:

- (I) Total annual herbaceous production may be estimated on the cropland Sample Unit and an approved cropland Reference Area using quadrats as described above and statistical analyses; or
- (II) Total annual herbaceous production may be evaluated on the cropland Sample Unit by harvesting the entire crop. The cropland harvest shall be compared to appropriate county production data for the same harvest year. Alternatively, total harvest may be compared to production from a nearby cropland unit for the same harvest year. This comparison shall not use any statistical test.

(J) Sampling for shrub density

The following information on shrubs located in shrub patches shall be collected to verify fulfillment of the applicable shrub goal or standard:

- (I) Shrub species list, including differentiation of full shrubs and subshrubs. Shrubs and subshrubs shall be counted if they are rooted inside the belt transect. Multiple-stemmed plants shall be counted as one individual;
- (II) The actual seed mix shall be identified for each shrub patch located on the map required in (C) above;
- (III) Areal extent of shrub patches, including distribution of shrub patches and mosaics, and what standards (shrub goal or standard) apply to each patch;
- (IV) Shrub and subshrub density, by species, shall be collected in 100-square meter belt transects whose dimensions are either 50 meters by two meters, or 100 meters by one meter. Any alternate plot size and shape must be approved by the Administrator prior to initiation of sampling; and
- (V) Sample size for shrub standard lands, including minimum sample size and sample adequacy calculations, shall be based on the criteria in Section 3 of this document describing statistical methods. If shrub goal lands are banked for the shrub standard, then requirements for sample adequacy and statistical tests apply. Sample size for shrub patches in shrub goal lands shall be determined on a case by case basis.

(K) Measuring tree replacement

Verification of the total number of trees shall:

- (I) Be determined using direct counts in the field or from aerial photographs;
- (II) Include verification that the performance standards for trees were met (Chapter 4 Section 2(d)(i)(I));
 - (1.) The number of postmining trees of the specified species or alternative volunteer species approved by the Administrator is equal to the number of premining trees,
 - (2.) Documentation that 80% of all planted trees were planted at least eight years prior to revegetation success evaluation, and
 - (3.) Documentation that all planted trees have been in place for at least two years.
- (III) Identify all trees by scientific binomial and common name; and
- (IV) Note if each species invaded or if it was listed in the approved reclamation plan.

(L) Sampling trees for reforestation

Tree replacement for the Forestry land use shall be evaluated using plotless methods, with a minimum of 20 randomly chosen sample points per vegetation community.

(M) Sampling for species diversity and composition

Species diversity and species composition shall be collected in each sample unit according to the approved sampling plan based on procedures in the approved permit commitments for the species diversity standard.

Section 2: Submittal of Information for Revegetation Success Evaluation

The applicant shall provide a report that references the approved sampling plan, includes the data collected during the revegetation success study and provides an evaluation of that data in accordance with the following requirements:

(A) **Methods description**

A methods section that includes a consolidated description of the methods, sampling dates, names of persons and company conducting the revegetation success study, and a statement that the study methods conformed to the approved sampling plan.

(B) **Maps**

A map for each year of sampling of the Sample Unit(s) and Reference Area(s) as described in Section 1(C) above.

(C) **Vegetation Descriptions**

Descriptions of each Sample Unit and each Reference Area as described in Section 1(D) above.

(D) **Seeding information**

Seeding dates and actual seed mixes for each Sample Unit.

(E) **Species inventory**

A species inventory in accordance with Section 1(E) above.

(F) **Acreage information**

A tabular summary of:

- (I) The acreage of each Sample Unit and each Reference area; and
- (II) The acreage of each mapped shrub goal and/or shrub standard patch.

(G) **Results of vegetation cover and production analyses using reference areas**

Presentation of the results for quantitative standards for absolute total vegetation cover and production using comparisons of Sample Units to Reference Areas shall include:

- (I) The results of the statistical analysis for each parameter and each Sample Unit, which shall include:
 - (1.) A description of the specific statistical analyses of each parameter for each Sample Unit per Section 3 of this document;

- (2.) A summary table, for each sample year, including:
 - a. The mean and standard deviation for each parameter within each Sample Unit and each Reference Area;
 - b. The critical value and calculated value of the test statistic for each parameter within each Sample Unit;
 - c. The sample size for each parameter and each Sample Unit and Reference Area; and
 - d. If sample adequacy is required per Section 3 of this document, the summary table will also include the calculated minimum sample size for sample adequacy for each parameter.
 - (3.) Sample calculations of the statistical tests and sample adequacy shall be provided; and
 - (4.) If t-tests were used for an analysis, verification that the assumptions of normal distribution and equal variances shall be provided.
- (II) Demonstrations that sampling to required minimum sample size, attainment of sample adequacy (if required), and statistical test results support achievement of the performance standards in Chapter 4, Section 2(d)(ii).

(H) Results of vegetation cover and production analyses using technical standards

Presentation of the results for the quantitative standards for total absolute vegetation cover and production using comparisons of Sample Units to technical standards (as an alternative to (G) above) shall include:

- (I) A description of the Technical Standard developed for each parameter and a reference to the Administrator's approval of the technical standard.
- (II) The results of the statistical analysis for each parameter and each Sample Unit, including:
 - (1.) A description of the statistical analyses used for each parameter for each Sample Unit;
 - (2.) A summary table, for each sample year, including:
 - a. The mean and standard deviation for each parameter within each Sample Unit;
 - b. The critical value and calculated value of the test statistic for each parameter within each Sample Unit;
 - c. The sample size for each parameter and each Sample Unit; and
 - d. If sample adequacy is required per Section 3 of this document, the summary table will also include the calculated minimum sample size for sample adequacy for each parameter.
 - (3.) Sample calculations for the statistical tests and sample adequacy shall be provided; and

- (4.) If t-tests were used for an analysis, verification that the assumptions of normal distribution and equal variances shall be provided.

(III) Demonstrations that sampling to required minimum sample size, attainment of sample adequacy (if required), and statistical test results support achievement of the performance standards in Chapter 4, Section 2(d)(ii).

(I) Results of the shrub density standard analysis

Presentation of results for the quantitative technical standard for Shrub density, when applicable, shall include:

- (I) A statement of the shrub density standard option approved for each Sample Unit, the actual shrub density standard for each Sample Unit, including the requirements for shrub density and shrub species composition, and the number of acres required to meet the shrub standard for each Sample Unit.
- (II) Presentation of the information required in Section 1(J) above.
- (III) The results of the statistical analysis for shrub density for each Sample Unit, which shall include:
 - (1.) A description of the statistical analyses used for each Sample Unit;
 - (2.) A summary table, for each sample year with the following information:
 - a. The mean and standard deviation for shrub density within each Sample Unit;
 - b. The critical value and calculated value of the test statistic for shrub density within each Sample Unit;
 - c. The sample size for each Sample Unit; and
 - d. If sample adequacy is required per Section 3 of this document, the summary table will also include the calculated minimum sample size for sample adequacy for shrub density.
 - (3.) Sample calculations for the statistical tests and sample adequacy shall be provided; and
 - (4.) If t-tests were used for an analysis, verification that the assumptions of normal distribution and equal variances shall be provided.
- (V) Discussion and summary table comparing the shrub density and shrub species composition standards to the shrub density and shrub species composition measured in the shrub standard patches.
- (VI) Demonstrations that sampling to required minimum sample size, attainment of sample adequacy, and statistical test results support achievement of the shrub standard for shrub density and species composition per Chapter 4, Section

2(d)(ii)(A)(II)(2.) and Appendix 4-2. This should include the number of acres, if any, banked toward other areas.

(J) Results of the shrub goal assessment

Presentation of results for the shrub goal, when applicable, shall include:

- (I) Presentation of the shrub goal information required in Section 1(J) above;
- (II) Presentation and discussion of a summary tabulation of the sample size and the shrub density mean value and standard deviation sampled across all mapped shrub patches in each Sample unit;
- (III) Statement of the required acreage of shrub patches per Chapter 4, Section 2(d)(ii)(A)(II)(1.) for each sample unit;
- (IV) Statement that the shrub restoration practices approved at the date of permanent reclamation of the sample unit were, in fact, applied to the sample unit; and
- (V) Statement whether the applicant intends to bank any acreage of shrub patches from each sample unit toward the shrub goal or standard:
 - (1.) No shrub goal acreage may be banked toward the shrub standard until cumulative accounting of acreage across all life-of-mine prospective shrub goal sample units demonstrates that excess goal acreage exists;
 - (2.) All prospective shrub goal acreage banked toward the shrub standard shall use the sampling methods and quantitative analyses required for the shrub standard described in Sections 1(I) and 2(I) above, and shall achieve the shrub standard density and composition performance standards; and
 - (3.) Each request to bank shrub goal acreage toward the shrub standard shall include a cumulative summary of the previously approved banked acreage and the dates of division approval of these requests.

(K) Results of the species diversity and composition assessment

Presentation of results for the verification of the permit-specific standards for species diversity and species composition shall include:

- (I) A reference to the permit commitments for the species and diversity standard approved by LQD;
- (II) A description of the sampling and any statistical analysis methods used to assess species diversity and composition;

- (III) For each sample year, a separate discussion and summary table demonstrating achievement of the species diversity and species composition standards; and
- (IV) Presentation of conclusions regarding achievement of the diversity performance standards.

(L) Results of the cropland standard analysis

Presentation of results for the cropland standard, when applicable, shall include:

- (I) A statement of the approved cropland standard;
- (II) Verification that the cropland standard was achieved using whole-field harvest comparisons shall include a summary table comparing the whole field harvest of the Sample Unit to whole-field harvest of the Reference Area or county data. Separate comparisons shall be presented for each harvest year;
- (III) Verification that the cropland standard was achieved using production harvest plots following methods described in Section 1 of this document, and requirements for submittal in Section 2(G)(I) and (II) above; and
- (IV) Presentation of conclusions regarding achievement of the cropland standard for each harvest year.

(M) Results of the tree replacement standard assessment

Presentation of results for the semi-quantitative standard for tree replacement, shall include:

- (I) A statement of the approved tree replacement standard for the Sample Unit; and
- (II) Demonstrations of achievement of the tree replacement standard, if applicable, as described in Section 1(K) above.

(N) Demonstration of mitigation wetlands restoration

Presentation of results for mitigation wetland restoration, if applicable, shall include:

- (I) A copy of the data submitted to the Army Corps of Engineers for verification of wetland restoration; and
- (II) A copy of the acceptance letter from the Army Corps of Engineers determining that wetland restoration was successful.

(O) Results of the reforestation standard analysis

Presentation of the results for successful reforestation, if applicable, shall include:

- (I) A description of the reclamation standards for each Sample Unit;
- (II) Verification that the quantity and quality of trees achieved the standard (Chapter 4, Section 2(d)(ii)(H)); and
- (III) Verification that the understory plant cover achieved the permit commitment.

(P) Results from fish & wildlife habitat, recreational, industrial/commercial or residential, and mitigation wetland analyses

Presentation of the results for verification of successful reclamation of the Fish and Wildlife Habitat, Recreational, Enhancement Wetland, and Industrial/Commercial and Residential land uses, if applicable, shall include:

- (I) A description of the permit-specific reclamation standards for each Sample Unit; and
- (II) Verification of achievement of the standards for each Sample Unit in a format approved by the Administrator.

(Q) Appendices

- (I) Photographs, if included, should have a complete legend, a unique identification number, and the location of each on a map (reference);
- (II) The raw data presented as one of the following: field data sheets, a spreadsheet containing the field data or another record approved by the Administrator; and
- (III) Computations and summaries of:
 - (1.) Absolute and relative percent species cover with species grouped by life form, and absolute percent total vegetation cover, rock cover, and litter cover, and
 - (2.) Total herbaceous production.

(IV) Literature citations

Section 3: Statistical Procedures for Revegetation Success Evaluation

This section provides coal operators with statistical procedures approved by LQD for revegetation success evaluation. There are two general categories of statistical tests for evaluating revegetation success. *One-sample tests* are used when the value of the reclamation is compared to a technical standard (e.g., the shrub density standard). *Two-sample tests* are used when the value of the reclamation is compared to the value of a reference, control, or comparison area.

Nonparametric reverse-null tests may be used as soon as the minimum sample size is obtained. These tests are the one-sample reverse-null sign test and the two-sample reverse-null Mann-Whitney test. The t-tests carry assumptions of normality and, for two-sample tests, equal variances. These assumptions must be evaluated before the t-tests may be used. In addition, sample adequacy must be obtained if the classic t-test approach is used.

The following procedures are detailed in this appendix:

(1) **One-sample Procedures (page 22)**

- 1.1 LQD Minimum Sample Sizes;
- 1.2 Normal Probability Plot / Shapiro-Francia Test;
- 1.3 One-sample Sample Adequacy Formula;
- 1.4 One-sample Sign Test of Reverse Null Hypothesis;
- 1.5 One-sample t-test of Classical Null Hypothesis; and
- 1.6 One-sample t-test of Reverse Null Hypothesis.

(2) **Two-sample Procedures (page 28)**

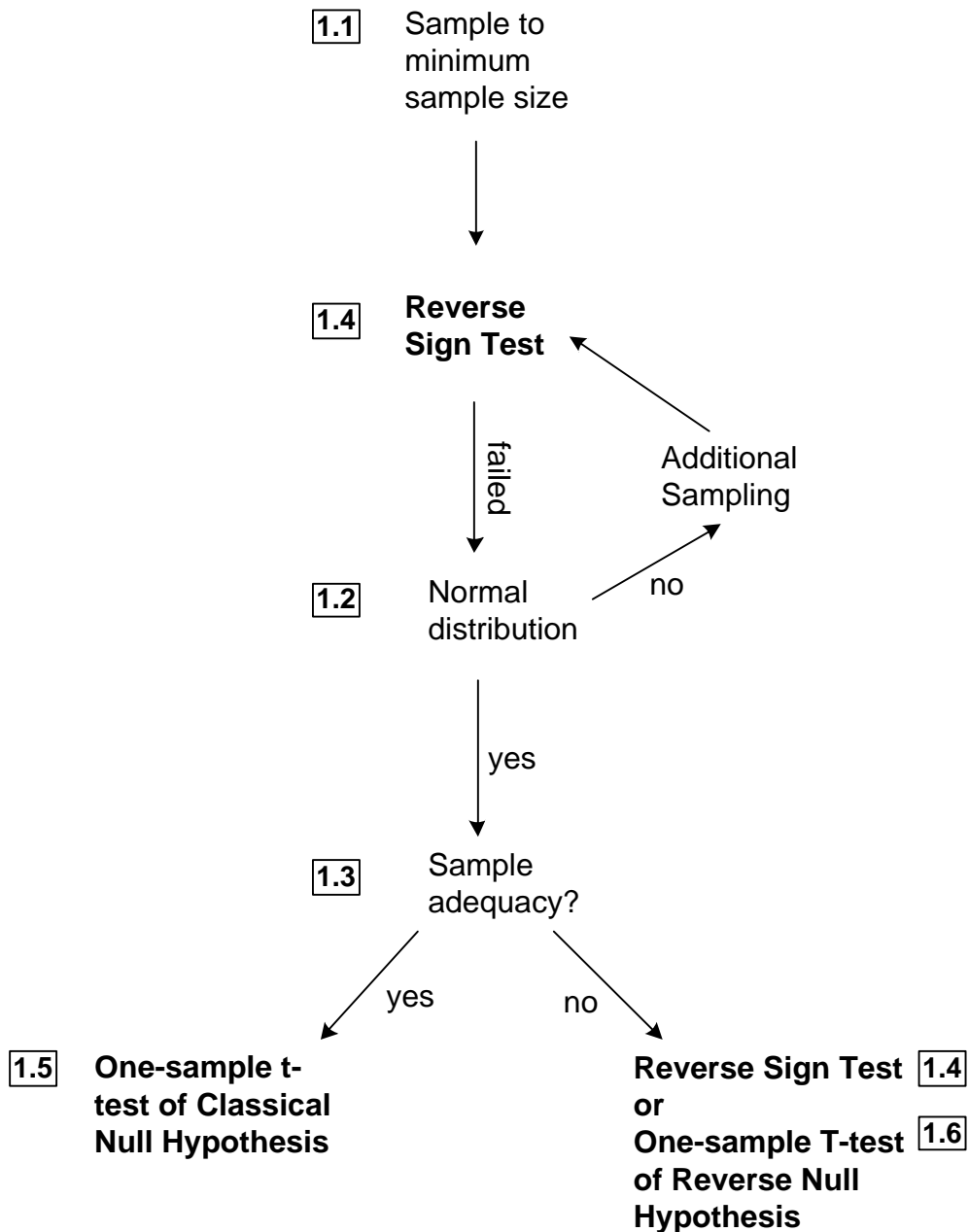
- 2.1 LQD Minimum Sample Sizes;
- 2.2 Two-sample Mann-Whitney Test of Reverse Null Hypothesis;
- 2.3 Normal Probability Plot / Shapiro-Francia Test;
- 2.4 Levene's Test for Homogeneity of Variance;
- 2.5 Two one-sided Sample Adequacy formulas with alpha adjustment;
- 2.6 Two-sample t-test of Classical Null Hypothesis;
- 2.7 Two-sample t-test of Reverse Null Hypothesis with Satterthwaite's Approximation; and
- 2.8 Two-sample t-test of Classical Null Hypothesis with Satterthwaite's Approximation.

Acceptable statistical procedures are determined using a flowchart for one- or two-sample data sets. Each flowchart contains a set of sequential questions to be answered "yes" or "no" by the user. Each pathway through the flowchart will end in a statistical procedure that is appropriate for that particular data set. There are no assumptions for the reverse null nonparametric tests. Therefore, the data may be tested with the nonparametric test after attaining the minimum sample size. If the nonparametric test does not support successful revegetation, the operator should follow the flowchart to determine alternative tests. The reverse null tests are more difficult tests to pass for successful revegetation. Therefore, if the test of the data fails the reverse null nonparametric test, the optimal test would be the classic t-test for one- or two-samples, provided the assumptions are met.

The critical value for a t-test of size alpha with left tail probability and df degrees freedom is specified by $t_{\alpha,df}$. The critical t value, $t_{1-\alpha,df}$, specifies a test of size alpha with right tail probability and df degrees of freedom. Similar definitions are used for the standard normal critical values: z_{α} and $z_{1-\alpha}$. Classical hypothesis t-tests are left tail tests comparing a test statistic with a negative critical value (t_{α} or z_{α}) while reverse null hypothesis t-tests are right tailed tests comparing a test statistic with a positive critical value ($t_{1-\alpha}$ or $z_{1-\alpha}$). Critical values of the Student's t and normal z distribution are in Table 1. The formulas use alpha equal to 0.10 and b equal to 0.90. In this document, N_{br} is the sample size of the revegetation, and N_{ref} is the sample size of the reference, comparison, or control area. Reference area refers to all types of native areas used for comparison against reclamation to assess revegetation success.

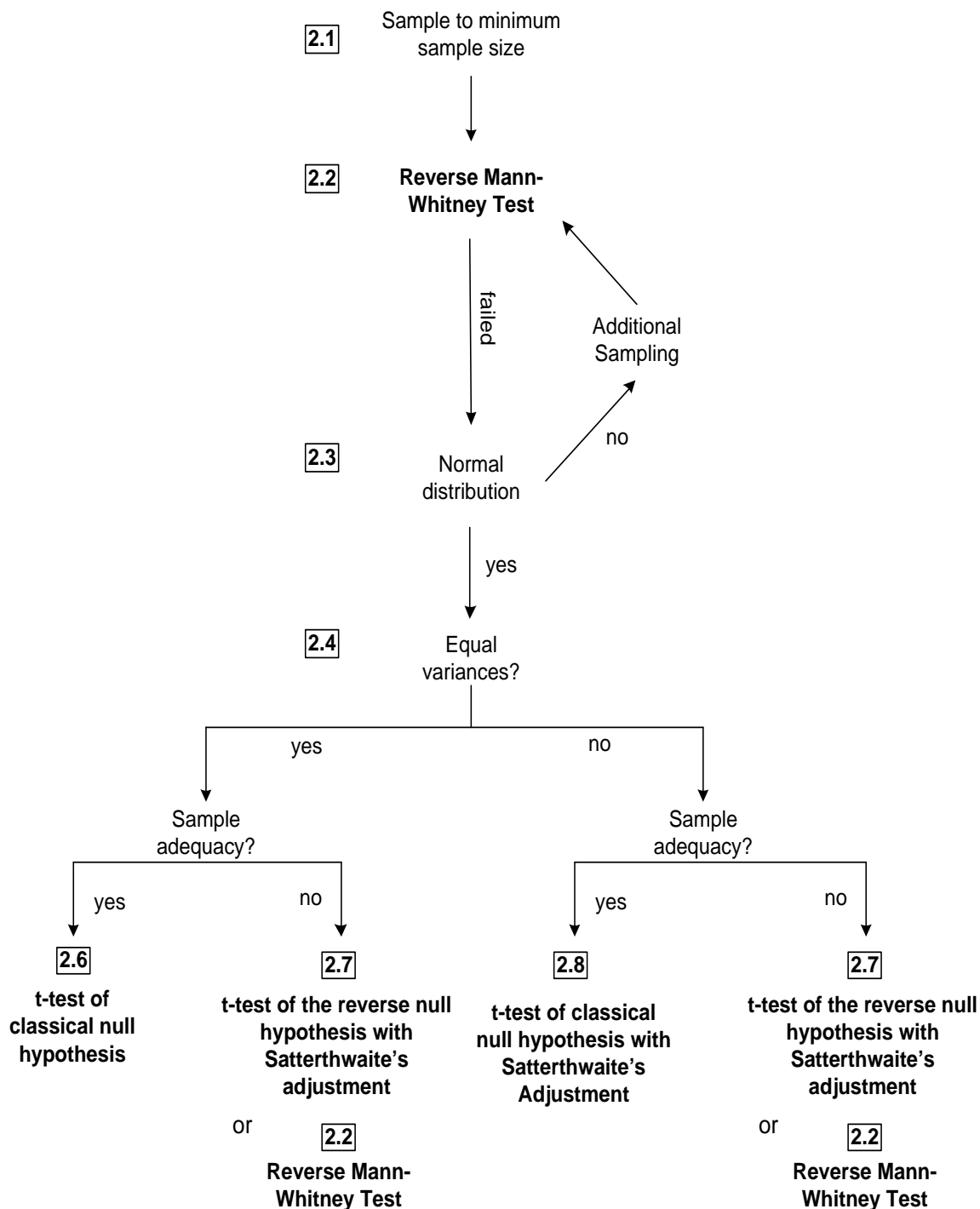
Flow Charts for Statistical Procedures

One-Sample Procedures for Comparison to a Technical Standard



Additional samples may be collected for re-evaluation of the data if the minimum sample size does not yield successful test results and/or sample adequacy.

Two-Sample Procedures for Comparison to a Reference, Control, or Comparison Area



Additional samples may be collected for re-evaluation of the data if the minimum sample size does not yield successful test results and/or sample adequacy.

Example Data and Calculations

The following dataset is used in the example calculations. The calculations of the mean, variance, standard deviation, and standard error used throughout this section and are shown here. When one-sample procedures are demonstrated, the revegetation (BR) sample data will be compared to a technical standard of 14.5. When two-sample procedures are demonstrated, the revegetation (BR) sample data will be compared to Reference (REF) data.

Observation number	BR data	REF data
1	14	24
2	23	15
3	8	9
4	25	10
5	17	19
6	20	28
7	28	7
8	22	11
9	18	9
10	10	10
11	14	14
12	21	22
13	9	14
14	31	26
15	20	21
Data Summaries		
Sum	280	239
Mean	18.67	15.93
Variance	46.24	47.35
Standard Deviation	6.80	6.88
Standard Error of Mean	1.76	1.78

The **mean** (\bar{X}_{br}) of these data is calculated as

$$\bar{X} = \frac{\sum_{i=1}^{n_{br}} x_i}{n_{br}}$$

For the revegetation sample data, the sum of the observations (numerator of mean formula) is 280 and the sample size is 15 (n_{br}), therefore the mean is 18.67:

$$\bar{X} = \frac{280}{15} = 18.67.$$

The **variance** (s_{br}^2) of these data is calculated as

$$s^2 = \frac{\sum_{i=1}^{n_{br}} (x_i - \bar{x})^2}{n - 1}.$$

For the revegetation sample data, the sum of the squared observations minus the mean (numerator of mean formula) is 647.33 and the sample size is 15 (n), therefore the variance is 46.24:

$$s^2 = \frac{647.33}{14} = 46.24.$$

The **standard deviation** (s_{br}) of these data is calculated as

$$s = \sqrt{s^2} = \sqrt{\frac{\sum_{i=1}^{n_{br}} (x_i - \bar{x})^2}{n_{br} - 1}}.$$

For the revegetation sample data, the variance is 46.23, therefore the standard deviation is 6.80:

$$s = \sqrt{46.24} = 6.80.$$

The **standard error of the mean** (SE_{br}) for these data is calculated as

$$SE = \frac{s}{\sqrt{n_{br}}}.$$

For the revegetation sample data, the standard deviation is 6.80 and the sample size is 15 (n), therefore the standard error of the mean is 1.76:

$$SE = \frac{6.80}{\sqrt{15}} = 1.76.$$

One-sample procedures

1.1. LQD Minimum Sample Sizes

Reclamation areas less than 10 acres:	10
Reclamation areas 10 or more acres:	20

1.2. Assessment of Normality in Dataset

Normality may be assessed using a Normal Probability Plot, the Shapiro-Francia Test described below, or by evaluating the distribution of the plotted data. If a t-test is used for revegetation success evaluation, the data must meet the assumption of a normal distribution.

Normal Probability Plot

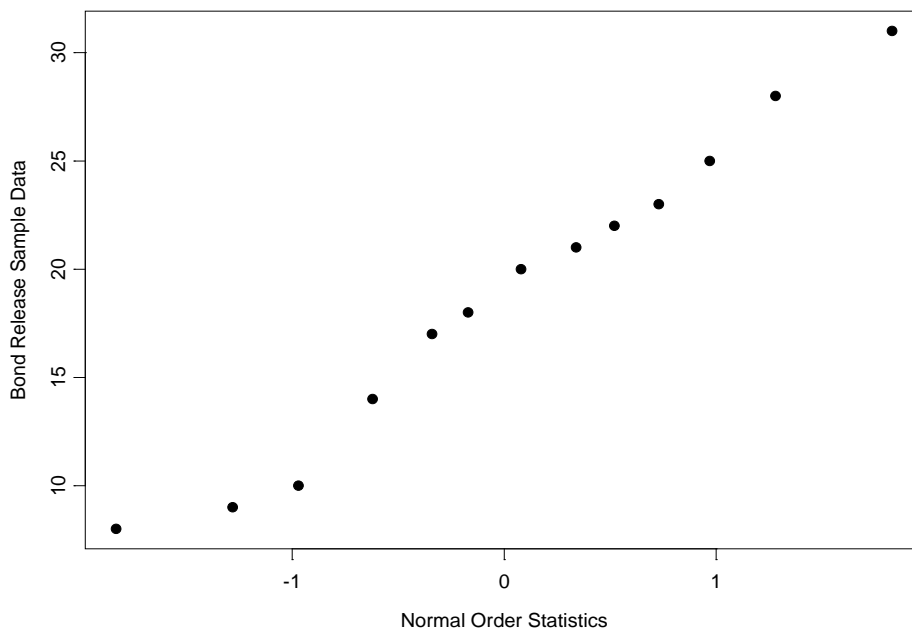
A normal probability plot is created by plotting the ordered sample data against a sample of the normal order statistics. The normal order statistics are calculated by adjusting the ranked data and calculating the inverse of a cumulative standard normal distribution:

$$p_i = (R_i - .5) / n$$

$$z_i = \Phi^{-1}(p_i)$$

where R_i are the ranks of the data, n is the sample size, p_i are the transformed ranks used for calculating the order statistics, Φ^{-1} is the inverse cumulative standard normal distribution function, z_i are the normal order statistics. The rank of an observation is its position among the set of observations when arranged from the smallest to largest. The order statistics represent the expected values of the data if they came from a normal distribution. If the data are normally distributed, the actual data will be close to the expected values and the plot will be a straight line. Extreme non-normality is present if the graph is visually very different from a straight line.

Example



The plot of the sample revegetation data is an approximately straight line indicating no serious departures from normality.

Shapiro-Francia Test

The Shapiro-Francia test with alpha equal to 0.01 is used to test if a data set is normally distributed. The null hypothesis for the Shapiro-Francia test states that the sample data are normally distributed. The test statistic is:

$$W' = \frac{(\sum_{i=1}^n b_i y_i)^2}{\sum_{i=1}^n (y_i - \bar{y})^2}$$

with

$$b_i = \frac{z_i}{(\sum_{i=1}^n z_i^2)^{\frac{1}{2}}}$$

where y_i is the sample data, \bar{y} is the sample mean, n is the sample size, and z_i is the order statistic corresponding to y_i . We reject the null hypothesis when the test statistic is small, i.e. the probability of observing the test statistic under the null hypothesis of normal data is small (less than 0.01). The null hypothesis is rejected for $W' <$ the critical value, the percentage points of the empirical W' distribution corresponding to a test of level 0.01 (Table 2). If we accept the null hypothesis of normality, then we conclude that the data are approximately normal and the regular t-tests give precise approximations for p-values. Assumptions for this test include independence of observations.

Example

Using the example data for the revegetation sample, the rank of an observation (R_i) is the position of the observation when the data are ordered from smallest to largest. The transformed rank (p_i) is the rank (R_i) minus 0.5 and divided by 15 (n). The order statistic (z_i) is the cumulative normal distribution quantiles of the p_i (can be calculated with the Microsoft excel function *norminv*($p_i,0,1$)). The order statistics are squared, summed and square rooted to obtain the denominator of the b_i 's. Each b_i is multiplied by the corresponding y_i , these numbers are summed and squared to obtain the numerator of the W' statistic. The mean of the raw data (\bar{y}) is subtracted from each y_i , squared and summed to obtain the denominator of the W' statistic. The numerator is divided by the denominator to obtain W' . The normal probability plot is obtained by plotting the y_i by the z_i .

Observation number	Bond Release Data	Ranked Bond Release Data	Transformed Ranks	Order Statistics	Squared Order Statistic	Order Statistic $(\sum z_i)^{1/2}$	Numerator of W'	Denominator of W'
	y_i	R_i	p_i	z_i	z_i^2	b_i	$b_i * y_i$	$(y_i - \bar{y})^2$
3	8	1	0.0333	-1.8339	3.3632	-0.4948	-3.9582	113.7780
13	9	2	0.1000	-1.2816	1.6424	-0.3458	-3.1118	93.4440
10	10	3	0.1667	-0.9674	0.9359	-0.2610	-2.6101	75.1110
1	14	4.5	0.2667	-0.6229	0.3880	-0.1681	-2.3529	21.7780
11	14	4.5	0.2667	-0.6229	0.3880	-0.1681	-2.3529	21.7780
5	17	6	0.3667	-0.3407	0.1161	-0.0919	-1.5626	2.7780
9	18	7	0.4333	-0.1679	0.0282	-0.0453	-0.8153	0.4440
6	20	8.5	0.5333	0.0837	0.0070	0.0226	0.4514	1.7780
15	20	8.5	0.5333	0.0837	0.0070	0.0226	0.4514	1.7780
12	21	10	0.6333	0.3407	0.1161	0.0919	1.9303	5.4440
8	22	11	0.7000	0.5244	0.2750	0.1415	3.1126	11.1110
2	23	12	0.7667	0.7279	0.5299	0.1964	4.5169	18.7780
4	25	13	0.8333	0.9674	0.9359	0.2610	6.5251	40.1110
7	28	14	0.9000	1.2816	1.6424	0.3458	9.6812	87.1110
14	31	15	0.9667	1.8339	3.3632	0.4948	15.3382	152.1110

$\bar{y} = 18.67$

$\sum z_i^2 = 13.74$

$\sum b_i * y_i = 25.24$

$\sum (y_i - \bar{y})^2 = 647.33$

$(\sum z_i^2)^{1/2} = 3.71$

$(\sum b_i * y_i)^2 = 637.22$

$$W' = \frac{637.22}{647.33} = 0.9844$$

The W' statistic is 0.9844. Using table 2, the lower critical value of the W' distribution is 0.919 (using sample size of 35). Since the test statistic, 0.9844, is greater than the lower critical value, 0.919, the null is retained, the data is normally distributed.

1.3. One-sample Sample Adequacy Formula

The number of samples required to ensure 90% confidence ($\alpha=0.10$) that the sample mean will be within 10% (d) of the true mean is:

$$n = \frac{(sz)^2}{(d\bar{x})^2}$$

where n is the number of sample points needed,

s is the sample standard deviation,

z is the critical value from the normal distribution with $\alpha/2$ in each tail (1.645),

d is the percentage of the sample mean that constitutes an acceptable difference between the sample mean and the true population mean (0.10), and

\bar{x} is the sample mean.

Example

Using the example data for the bond release sample, the standard deviation is 6.80, the z critical value with 0.05 probability in each tail is 1.645, the d is 0.10, and the sample mean is 18.67. The number of samples required to reach sample adequacy is calculated as:

$$n = \frac{(6.8 * 1.645)^2}{(0.10 * 18.67)^2} = 35.89 \text{ or } 36.$$

1.4. Nonparametric One-sample Sign Test of Reverse Null Hypothesis

The reverse-null sign test is appropriate for comparing a revegetation community parameter to a technical standard.

There are no assumptions that need to be met for this test. The reverse null hypothesis for the nonparametric sign test is that the values of a given revegetation parameter is equal to or less than 90% of the technical standard.

$$H_o : br \text{ values} - 0.9 * tech.st. \leq 0$$

$$H_a : br \text{ values} - 0.9 * tech.st. > 0$$

Evidence in support of revegetation success is obtained when the null hypothesis is rejected.

With small sample sizes (≤ 20) there are exact probabilities of observing the number of minuses (M) when the null is true. Critical values for a test corresponding to the exact probabilities are in Table 3.

The approximate test statistic for sample sizes beyond the tables is:

$$Z = \frac{M + 0.5 - 0.5n}{0.5\sqrt{n}}$$

where **M** is the number of minuses when 0.9*technical standard is subtracted from every observation, and **n** is the sample size.

The standard is met when the one-sided null hypothesis is rejected: the test statistic is less than z_{α} (Table 1). The conclusion is that the values from the revegetation tend to be greater than 90% of the technical standard.

Example

Using the example data for the revegetation (BR) sample, the sample size is 15, and the number of observations which have negative values (minuses) when 90% of the technical standard (14.5 is the technical standard used throughout the appendix, $0.90 \cdot 14.5 = 13.05$) is subtracted from each observation is 3 (observation numbers 3, 10, and 13). Using table 3, the lower critical value corresponding to a sample size of 15 is 4. Since the observed number of minuses, 3, is less than or equal to the lower critical value, null is rejected, the bond release values tend to be statistically greater than 90% of the technical standard.

For purposes of this example, the calculated approximate test statistic is -2.07:

$$Z = \frac{3 + 0.5 - (0.5 \cdot 15)}{0.5 \sqrt{15}} = -2.07$$

The test statistic is compared to a z critical value with 0.10 probability ($Z_{0.10}$) in the left tail (-1.28). Since the calculated z-statistic, -2.07, is less than the z critical value, -1.28, the null is rejected, the revegetation values tend to be statistically greater than 90% of the technical standard.

1.5. One-sample t-test of Classical Null Hypothesis

The null hypothesis for the classical one-sample t-test is that the mean of a given vegetation parameter (μ_{br}) is equal to or greater than a technical standard.

$$H_o : \mu_{br} - tech.st. \geq 0$$

$$H_a : \mu_{br} - tech.st. < 0$$

Evidence in support of revegetation success is obtained when the null hypothesis is not rejected (the null hypothesis is accepted). Assumptions for this test include sample adequacy, approximate normality and independence of observations. The test statistic is:

$$t = \frac{\bar{x} - tech.st.}{(s / \sqrt{n})}$$

where \bar{x} is the sample mean,
tech.st. is the technical standard,
s is the sample standard deviation, and
n is the sample size.

The standard is met when the one-sided null hypothesis is accepted: the test statistic is greater than $t_{\alpha, n-1}$. The conclusion is that the mean of the revegetation parameter is equal to or greater than the technical standard.

Example

Using the example data for the revegetation (BR), the sample mean is 18.67, the standard deviation is 6.80,

the sample size is 15, and the technical standard is 14.5 (this is the technical standard used throughout the appendix). The calculated test statistic is 2.38:

$$t = \frac{18.67 - 14.5}{(6.8/\sqrt{15})} = 2.38$$

The test statistic is compared to a t critical value with 0.10 probability ($t_{0.10}$) in the left tail and 14 degrees of freedom (-1.35). Since the calculated t-statistic, 2.38, is greater than the t critical value, -1.35, the null is retained, the revegetation mean is statistically greater than or equal to the technical standard. Revegetation success is supported by the data. Note that this is an example, since the data did not meet sample adequacy this test would not have been performed.

1.6. One-sample t-test of Reverse Null Hypothesis

The reverse null hypothesis is that the mean of a given vegetation parameter (μ_{br}) is equal to or less than 90% of the technical standard.

$$H_o : \mu_{br} - 0.9 * tech.st. \leq 0$$

$$H_a : \mu_{br} - 0.9 * tech.st. > 0$$

Evidence in support of revegetation success is obtained when the null hypothesis is rejected. Assumptions for this test include approximate normality and independence of observations. The test statistic is:

$$t = \frac{\bar{x} - 0.9 * tech.st.}{(s/\sqrt{n})}$$

where \bar{x} is the sample mean,

tech.st. is the technical standard,

s is the sample standard deviation, and

n is the sample size.

The standard is met when the one-sided null hypothesis is rejected: the test statistic is greater than $t_{1-\alpha, n-1}$. The conclusion is that the mean for the revegetation is greater than 90% of the technical standard.

Example

Using the example data for the revegetation (BR) sample, the sample mean is 18.67, the standard deviation is 6.80, the sample size is 15, and the technical standard is 16. The calculated test statistic is 3.20:

$$t = \frac{18.67 - (.9 * 14.5)}{(6.8/\sqrt{15})} = 3.20$$

The test statistic is compared to a t critical value with 0.10 probability ($t_{0.90}$) in the right tail and 14 degrees of freedom (1.35). Since the calculated t-statistic, 3.20, is greater than the t critical value, 1.35, the null is rejected, the bond release mean is statistically greater than 90% of the technical standard. Revegetation success is supported by the data.

Two-sample procedures

2.1. LQD Minimum Sample Sizes

Reclamation and reference areas less than 10 acres: 10
Reclamation and Reference areas 10 or more acres: 20

2.2. Two-sample Mann-Whitney Test of Reverse Null Hypothesis

The reverse null hypothesis for the Mann-Whitney test is that the revegetation values tend to be less than 90% of the reference values.

H_o : revegetation values ≤ 0.9 *reference values

H_a : revegetation values > 0.9 *reference values

Evidence in support of revegetation success is obtained when the null hypothesis is rejected.

There are no assumptions that need to be met for this test.

With small sample sizes (≤ 20) there are exact probabilities of observing the sum of the ranks when the null is true. Critical values for a test corresponding to the exact probabilities are in Table 4. The approximate test statistic for sample sizes beyond Table 4 is:

$$Z = \frac{\sum_{i=1}^n R(br_i) + \frac{1}{2} - n_{br} \frac{(n_{br} + n_{ref} + 1)}{2}}{\sqrt{\frac{n_{br} n_{ref} (n_{br} + n_{ref} + 1)}{12}}}$$

where $\mathbf{R}(br_i)$ are the ranks of the observations in the revegetation sample, n_{br} is the revegetation area sample size, and n_{ref} is the reference area sample size.

Before ranking the entire dataset, every observation in the reference sample is multiplied by 0.9. Ranks are determined by sorting the combined observations from both data sets from smallest to largest value. Ranks are then assigned starting with 1, and ending with the combined sample size ($n_{ref} + n_{br}$). Equal values are given equal ranks, determined by calculating the average of the summed rank values. For example, two equal values might be ranked 9 and 10, so each would be assigned a rank of 9.5. The ranks are then summed for the revegetation area ($R(br_i)$).

Revegetation is successful when the one-sided null hypothesis is rejected: the test statistic is greater than $Z_{1-\alpha}$. The conclusion is that the values from the revegetation distribution tend to be greater than 90% of the values from the reference distribution.

Example

Using the example data, the revegetation sample size is 15 and the reference area sample size is 15. The sum of the ranks of the revegetation sample is 274. Using table 4, the upper critical value corresponding to sample sizes of 15 and 15, is 264. Since the observed sum of the ranks, 274, is greater than 264, the null is rejected, the revegetation values tend to be statistically greater than 90% of the reference values.

For example purposes, the z-statistic is 1.74:

$$Z = \frac{274 + \frac{1}{2} - (15 * \frac{(15 + 15 + 1)}{2})}{\sqrt{\frac{15 * 15 * (15 + 15 + 1)}{12}}} = 1.74.$$

The z-statistic is compared to a z critical value with 0.10 probability ($Z_{0.90}$) in the right tail (1.28). Since the calculated z-statistic, 1.74, is greater than the z critical value, 1.28, the null is rejected, the reclamation values tend to be statistically greater than 90% of the reference values. Revegetation success is supported by the data.

2.3. Assessment of Normality in Dataset

Use procedures presented in section 1.2 of this appendix for each dataset.

2.4. Levene's Test for Homogeneity of Variance

Levene's test is used to determine if the variances in two sample populations are "substantially" different, which would invalidate the two-sample t-test for differences in means. The two-sample t-test for differences in means is precise (robust) for approximately normal data unless the variances are quite different. For this reason, Levene's test of equal variances is conducted at the conservative level of alpha = 0.01 (two-sided test), making the test fairly easy to pass.

The null hypothesis for Levene's test states that the variance of the revegetation sample data is equal to the variance of the reference area sample data.

$$H_o : \sigma_{br}^2 = \sigma_{ref}^2$$

$$H_a : \sigma_{br}^2 \neq \sigma_{ref}^2$$

The test statistic is the two-sample t-statistic for the absolute deviations of every observation from the sample mean. For each sample, calculate the absolute deviations for each observation as

$$Z_{br_i} = |y_{br_i} - \bar{y}_{br}|$$

$$Z_{ref_i} = |y_{ref_i} - \bar{y}_{ref}|$$

and test for differences between the mean Z_{br} and mean Z_{ref} using the two-sample t-test (Subsection 2.6).

The variances are unequal if the null hypothesis is rejected: the test statistic is less than $t_{.005, n_{br} + n_{ref} - 2}$ or greater than $t_{0.995, n_{br} + n_{ref} - 2}$. If variances are unequal, then the Satterthwaite's adjustment must be used for the two-sample t-test.

Example

Using the example data, the mean for the revegetation data (18.67) was subtracted from each bond release observation and the mean for the reference area data (15.93) was subtracted from each reference area observation to obtain the absolute difference for each observation. The mean of the absolute differences for the bond release data was 5.42 with a standard deviation of 3.84. The mean of the absolute differences for the reference data was 5.92 with a standard deviation of 3.13. The two-sample t-test statistic is -0.39:

$$t = \frac{5.42 - 5.92}{\sqrt{\frac{(14 * 3.84^2) + (14 * 3.13^2)}{28} * (\frac{1}{15} + \frac{1}{15})}} = -0.39.$$

The two-sided t-statistic is compared to t critical values with alpha equal to 0.005 and 0.995 and 28 degrees of freedom (-2.76 and 2.76, Table 1). Since the calculated t-statistic, -0.39, is not less than -2.76 and not greater than 2.76, the null is retained and the conclusion is that the variances are not significantly different.

2.5. Two-sample Sample Adequacy formulas with alpha adjustment

In all two-sample situations, use the one-sample sample adequacy formula for each sample (revegetation and reference) with an adjustment to the alpha level to account for the fact that two samples are to be compared. This may result in different sample sizes for each sample, with more samples required for the more variable sample. The number of samples for each area required to ensure 90% confidence ($\alpha = 0.10$) that each sample mean will be within 10% (d) of the true mean simultaneously is:

$$n = \frac{(sz)^2}{(d\bar{x})^2}$$

where **n** is the number of sample points needed,

s is the sample standard deviation,

z is the critical value from the normal distribution with .025 in each tail (1.96),

d is the percentage of the sample mean that constitutes an acceptable difference between the sample mean and the true population mean (0.10), and

\bar{x} is the sample mean.

Example

Revegetation area sample adequacy

Using the example data for the revegetation sample, the standard deviation is 6.80, the z critical value with 0.025 probability in each tail is 1.96, the d is 0.10, and the sample mean is 18.67. The number of samples required to reach sample adequacy is calculated as:

$$n = \frac{(6.80 * 1.96)^2}{(0.10 * 18.67)^2} = 50.96 \text{ or } 51.$$

Reference area sample adequacy

Using the example data for the reference area sample, the standard deviation is 6.88, the z critical value with 0.025 probability in each tail is 1.96, the d is 0.10, and the sample mean is 15.93. The number of samples required to reach sample adequacy is calculated as:

$$n = \frac{(6.88 * 1.96)^2}{(0.10 * 15.93)^2} = 71.66 \text{ or } 72.$$

2.6. Two-sample t-test of Classical Null Hypothesis

The one-sided null hypothesis for the classical two-sample test is that the mean of a reclamation vegetation parameter (μ_{br}) is equal to or greater than the mean in the reference area (μ_{ref}).

$$H_o : \mu_{br} - \mu_{ref} \geq 0$$

$$H_a : \mu_{br} - \mu_{ref} < 0$$

Evidence in support of revegetation success is obtained when the null hypothesis is accepted.

Assumptions for this test include sample adequacy, approximate normality of both data sets, independence of observations, and approximate equality of variances between the two groups.

The test statistic is:

$$t = \frac{\bar{x}_{br} - \bar{x}_{ref}}{\sqrt{\frac{(n_{br} - 1)s_{br}^2 + (n_{ref} - 1)s_{ref}^2}{(n_{br} + n_{ref} - 2)} \left(\frac{1}{n_{br}} + \frac{1}{n_{ref}}\right)}}$$

where \bar{x}_{br} is the revegetation sample mean,

\bar{x}_{ref} is the reference area sample mean,

s_{br}^2 is the revegetation area variance,

s_{ref}^2 is the reference area variance,

n_{br} is the revegetation area sample size, and

n_{ref} is the reference area sample size.

Revegetation is successful when the one-sided null hypothesis is not rejected (accepted): the test statistic is greater than $t_{\alpha, n_{br} + n_{ref} - 2}$.

Example

Using the example data, the revegetation sample mean is 18.67, the variance is 46.24 and the sample size is 15. The reference area sample mean is 15.93, the variance is 47.35 and the sample size is 15. The t-statistic is 1.10:

$$t = \frac{18.67 - 15.93}{\sqrt{\frac{(14 * 46.24) + (14 * 47.35)}{28} * \left(\frac{1}{15} + \frac{1}{15}\right)}} = 1.10.$$

The t-statistic is compared to a t critical value with alpha equal to 0.10 and 28 degrees of freedom (-1.31). Since the calculated t-statistic, 1.10, is greater than the t critical value, -1.31, the null is retained, the revegetation mean is statistically greater than or equal to the reference mean. Note that this is an example, since the data did not meet sample adequacy this test would not have been performed.

2.7. Two-sample t-test of Reverse Null Hypothesis with Satterthwaite's Adjustment

Satterthwaite's adjustment is used in the two-sample t-test of reverse null hypothesis because the b in the hypothesis implies that the variances will not be equal (i.e., when the values in the reference set are multiplied by b then the standard deviation is multiplied by b and the variance is multiplied by b²).

The reverse null hypothesis states the mean of a given vegetation parameter (μ_{br}) is equal to or less than 90% of the mean on the reference area (μ_{ref}).

$$H_o : \mu_{br} - 0.9 * \mu_{ref} \leq 0$$

$$H_a : \mu_{br} - 0.9 * \mu_{ref} > 0$$

Evidence in support of revegetation success is obtained when the null hypothesis is rejected.

Assumptions for this test include approximate normality and independence of observations between the two groups. This test is used when the variances are not equal and the data sets have not met sample adequacy, and is an alternative to the nonparametric test.

The test statistic is:

$$t = \frac{\bar{x}_{br} - 0.9\bar{x}_{ref}}{\sqrt{\frac{s_{br}^2}{n_{br}} + \frac{(0.9 * s_{ref})^2}{n_{ref}}}}$$

and the degrees of freedom are:

$$df_{sat} = \frac{SE_{sat}^4}{\left(\frac{SE_{br}^4}{n_{br} - 1} + \frac{(0.9 * SE_{ref})^4}{n_{ref} - 1}\right)}$$

where \bar{x}_{br} is the revegetation sample mean,

\bar{x}_{ref} is the reference area sample mean,

s_{br}^2 is the revegetation sample variance,

s_{ref}^2 is the reference sample variance,

SE_{br} is the revegetation area standard error of the mean,

SE_{ref} is the reference area standard error of the mean,

SE_{sat} is the denominator of the t-statistic,

n_{br} is the revegetation area sample size, and

n_{ref} is the reference area sample size.

Revegetation is successful when the one-sided null hypothesis is rejected: the test statistic is greater than $t_{1-\alpha, df_{sat}}$.

The conclusion is that the revegetation mean is greater than 90% of the reference mean.

Example

Using the example data, the revegetation sample mean is 18.67, the variance is 46.24 and the sample size is 15. The reference sample mean is 15.93, the variance is 47.35 and the sample size is 15. The t-statistic is 1.82:

$$t = \frac{18.67 - (.9 * 15.93)}{\sqrt{\frac{46.24}{15} + \frac{(.81 * 47.35)}{15}}} = 1.82$$

The degrees of freedom are:

$$df_{sat} = \frac{\left(\sqrt{\frac{46.24}{15} + \frac{(.81 * 47.35)}{15}}\right)^4}{\left(\frac{1.76^4}{14} + \frac{(.9 * 1.78)^4}{14}\right)} = 27.89 \text{ or } 28$$

The t-statistic is compared to a t critical value with 0.10 probability in the right tail ($t_{0.90}$) and 28 degrees of freedom (1.31). Since the calculated t-statistic, 1.82, is greater than the t critical value, 1.31, the null is rejected, the revegetation mean is statistically greater than or equal to 90% of the reference mean.

Revegetation success is supported by the data.

2.8. Two-sample t-test of Classical Null Hypothesis with Satterthwaite's Adjustment

Satterthwaite's adjustment is used in the standard t-test when Levene's test results in the conclusion that the variances of the two data sets are not equal at the alpha equal to 0.01 level of significance.

The one-sided null hypothesis for the classical two-sample t-test is that the mean of a given vegetation parameter (μ_{br}) is equal to or greater than the mean in the reference area (μ_{ref}).

$$H_o : \mu_{br} - \mu_{ref} \geq 0$$

$$H_a : \mu_{br} - \mu_{ref} < 0$$

Evidence in support of revegetation success is obtained when the null hypothesis is not rejected.

Assumptions for this test include sample adequacy, approximate normality, and independence of observations. The test statistic is:

$$t = \frac{\bar{x}_{br} - \bar{x}_{ref}}{\sqrt{\frac{s_{br}^2}{n_{br}} + \frac{s_{ref}^2}{n_{ref}}}}$$

and the degrees of freedom are:

$$df_{sat} = \frac{SE_{sat}^4}{\left(\frac{SE_{br}^4}{n_{br} - 1} + \frac{SE_{ref}^4}{n_{ref} - 1}\right)}$$

where \bar{x}_{br} is the revegetation sample mean,

\bar{x}_{ref} is the reference area sample mean,

s_{br}^2 is the revegetation sample variance,

s_{ref}^2 is the reference area sample variance,

SE_{br} is the revegetation area standard error,

SE_{ref} is the reference area standard error,

SE_{sat} is the denominator of the t-statistic,

n_{br} is the revegetation area sample size, and

n_{ref} is the reference area sample size.

Revegetation is successful when the one-sided null hypothesis is not rejected (accepted): the test statistic is greater than $t_{\alpha, df_{sat}}$. The conclusion is that the revegetation mean is greater than the reference mean.

Example

Using the example data, the bond release sample mean is 18.67, the variance is 46.24 and the sample size is 15. The reference sample mean is 15.93, the variance is 47.35 and the sample size is 15. The t-statistic is 1.10:

$$t = \frac{18.67 - 15.93}{\sqrt{\frac{46.24}{15} + \frac{47.35}{15}}} = 1.10.$$

The degrees of freedom are:

$$df_{sat} = \frac{\left(\sqrt{\frac{46.24}{15} + \frac{47.35}{15}}\right)^4}{\left(\frac{1.76^4}{14} + \frac{1.78^4}{14}\right)} = 27.99 \text{ or } 28.$$

The t-statistic is compared to a t critical value with 0.10 probability in the left tail ($t_{0,10}$) and 28 degrees of freedom (-1.31). Since the calculated t-statistic, 1.10, is greater than the t critical value, -1.31, the null is retained, the bond release mean is statistically greater than or equal to the reference mean.

Table 2. Lower critical values of the empirical percentage points of the approximate W' test from Shapiro and Francia (1972). When the calculated W' test statistic is less than the critical value, the null hypothesis of normal data is rejected.

Bond Release Sample Size	Lower critical value for test with alpha 0.01
35	0.919
50	0.935
51	0.935
53	0.938
55	0.940
57	0.944
59	0.945
61	0.947
63	0.947
65	0.948
67	0.950
69	0.951
71	0.953
73	0.956
75	0.956
77	0.957
79	0.957
81	0.958
83	0.960
85	0.961
87	0.961
89	0.961
91	0.962
93	0.963
95	0.965
97	0.965
99	0.967

Table 3. Lower critical values of the exact binomial probabilities for the reverse sign test. When the number of minuses is less than or equal to the lower critical value, the null hypothesis is rejected at the alpha equal to 0.10 level.

Bond Release Sample Size	M
5	0
6	0
7	1
8	1
9	2
10	2
11	2
12	3
13	3
14	4
15	4
16	4
17	5
18	5
19	6
20	6

Table 4. Upper critical values of the exact binomial probabilities for the reverse Mann-Whitney test. When the sum of the ranks in the bond release dataset is greater than or equal to the upper critical value, the null hypothesis is rejected at the alpha equal to 0.10 level.

Bond Release Sample Size	Reference sample size															
	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
5	34	37	41	44	47	51	54	57	61	64	67	71	74	77	81	84
6	43	47	51	55	59	63	67	71	75	79	83	87	91	94	98	102
7	54	58	63	67	72	76	81	85	90	94	99	103	108	112	117	121
8	65	70	75	80	85	91	96	101	106	111	116	121	126	131	136	141
9	77	83	89	94	100	106	112	117	123	129	134	140	145	151	157	162
10	91	97	103	110	116	122	128	135	141	147	153	160	166	172	178	184
11	105	112	119	126	133	139	146	153	160	167	173	180	187	194	201	207
12	120	128	135	143	150	158	165	172	180	187	194	202	209	216	224	231
13	137	145	153	161	169	177	185	193	201	209	217	224	232	240	248	256
14	154	163	171	180	189	197	206	214	223	231	240	248	257	265	273	282
15	172	182	191	200	209	218	227	236	246	255	264	273	282	291	300	309
16	192	202	211	221	231	241	250	260	269	279	289	298	308	317	327	336
17	212	223	233	243	253	264	274	284	294	305	315	325	335	345	355	365
18	233	244	255	266	277	288	299	309	320	331	342	352	363	374	384	395
19	256	267	279	290	302	313	325	336	347	358	370	381	392	403	415	426
20	279	291	303	315	327	339	351	363	375	387	399	410	422	434	446	458